Homework # 1

11.3 Calculate the energies for the Bohr orbits with $n = 2$ and 3 for atomic hydrogen.

The energy for a Bohr orbit is

$$E_n = -\frac{m_e e^4}{8h^2\epsilon_0^2}\frac{1}{n^2}$$

The collection of constants on the right side of the equation may be expressed in terms of the Rydberg constant:

$$R_H = \frac{m_e e^4}{8ch^3\epsilon_0^2}$$

$$\frac{m_e e^4}{8h^2\epsilon_0^2} = R_H ch$$

$$= \left(109737.31534\ \text{cm}^{-1}\right)\left(2.99792458 \times 10^{10}\ \text{cm s}^{-1}\right)\left(6.626075 \times 10^{-34}\ \text{J s}\right)$$

$$= 2.1798739 \times 10^{-18}\ \text{J}$$

Therefore,

$$E_n = -\left(2.1798739 \times 10^{-18}\ \text{J}\right)\frac{1}{n^2}$$

For the Bohr orbits with $n = 2$ and 3,

$$E_2 = -\left(2.1798739 \times 10^{-18}\ \text{J}\right)\frac{1}{4} = -5.449685 \times 10^{-19}\ \text{J}$$

$$E_3 = -\left(2.1798739 \times 10^{-18}\ \text{J}\right)\frac{1}{9} = -2.422082 \times 10^{-19}\ \text{J}$$

11.5 What are the wavelengths associated with (a) an electron moving at $1.50 \times 10^8\ \text{cm s}^{-1}$, and (b) a 60-g tennis ball moving at $1500\ \text{cm s}^{-1}$?

The wavelengths can be calculated using

$$\lambda = \frac{h}{mv}$$

(a)
$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J s}}{\left(9.109 \times 10^{-31}\ \text{kg}\right)\left(1.50 \times 10^6\ \text{m s}^{-1}\right)} = 4.85 \times 10^{-10}\ \text{m} = 4.85\ \text{Å}$$

(b)
$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J s}}{\left(60 \times 10^{-3}\ \text{kg}\right)\left(1500 \times 10^{-2}\ \text{m}\right)} = 7.36 \times 10^{-34}\ \text{m}$$

The wavelength of the tennis ball is too small to be of practical significance, whereas the wavelength of the electron is too large to be ignored.

11.15 An electron in an excited state in a hydrogen atom can return to the ground state in two different ways: first, via a direct transition in which a photon of wavelength λ_1 is emitted and second, via an intermediate excited state reached by the emission of a photon of wavelength λ_2. This intermediate excited state then decays to the ground state by emitting another photon of wavelength λ_3. Derive an equation that relates λ_1 to λ_2 and λ_3.

The energy of the photon of wavelength λ_1, E_1, equals the sum of the energy of the photon of wavelength λ_2, E_2, and the energy of the photon of wavelength λ_3, E_3. That is,

$$E_1 = E_2 + E_3$$

$$\frac{hc}{\lambda_1} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_3}$$

$$\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$$

11.19 Scientists have found interstellar hydrogen atoms with quantum number n in the hundreds. Calculate the wavelength of light emitted when a hydrogen atom undergoes a transition from $n = 236$ to $n = 235$. In what region of the electromagnetic spectrum does this wavelength fall?

The wavenumber of the emitted radiation is

$$\tilde{\nu} = \left(109737\ \text{cm}^{-1}\right)\left|\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)\right|$$

$$= \left(109737\ \text{cm}^{-1}\right)\left|\left(\frac{1}{236^2} - \frac{1}{235^2}\right)\right| = 1.680406 \times 10^{-2}\ \text{cm}^{-1}$$

Therefore, the wavelength of this radiation is

$$\lambda = \frac{1}{\tilde{\nu}} = \frac{1}{1.680406 \times 10^{-2}\ \text{cm}^{-1}} = 59.5094\ \text{cm}$$

This wavelength is in the radio frequency region.

11.25 What is the probability of locating a particle in a one-dimensional box between $L/4$ and $3L/4$, where L is the length of the box? Assume the particle to be in the lowest level.

The probability is

$$P = \int_{L/4}^{3L/4} \psi^2 \, dx$$

$$= \int \left(\frac{2}{L}\right) \sin^2 \frac{\pi}{L} x \, dx$$

$$= \frac{2}{L} \left(\frac{x}{2} - \frac{\sin \frac{2\pi x}{L}}{\frac{4\pi}{L}} \right)_{L/4}^{3L/4}$$

$$= \frac{2}{L} \left[\left(\frac{3L}{8} - \frac{\sin \frac{3\pi}{2}}{\frac{4\pi}{L}} \right) - \left(\frac{L}{8} - \frac{\sin \frac{\pi}{2}}{\frac{4\pi}{L}} \right) \right]$$

$$= \frac{2}{L} \left[\left(\frac{3L}{8} + \frac{L}{4\pi} \right) - \left(\frac{L}{8} - \frac{L}{4\pi} \right) \right]$$

$$= \frac{2}{L} \left(\frac{L}{4} + \frac{L}{2\pi} \right)$$

$$= \frac{1}{2} + \frac{1}{\pi}$$

$$= 0.82$$

11.27 An important property of the wave functions of the particle in a one-dimensional box is that they are orthogonal; that is,

$$\int_0^L \psi_n \psi_m \, dx = 0 \qquad m \neq n$$

Prove this statement using ψ_1 and ψ_2 and Equation 11.23.

Let $m = 1$ and $n = 2$.

$$\int_0^L \psi_2 \psi_1 \, dx = \frac{2}{L} \int_0^L \sin \frac{2\pi x}{L} \sin \frac{\pi x}{L} \, dx$$

A table of integrals gives, for $a^2 \neq b^2$,

$$\int \sin ax \ \sin bx \, dx = \frac{\sin (a-b)\, x}{2\,(a-b)} - \frac{\sin (a+b)\, x}{2\,(a+b)}$$

Setting $a = 2\pi/L$ and $b = \pi/L$,

$$\int_0^L \psi_2 \psi_1 \, dx = \frac{2}{L} \left[\frac{\sin \frac{\pi x}{L}}{\frac{2\pi}{L}} - \frac{\sin \frac{3\pi x}{L}}{\frac{6\pi}{L}} \right]_0^L$$

$$= \frac{2}{L} \left[\left(\frac{\sin \pi}{\frac{2\pi}{L}} - \frac{\sin 3\pi}{\frac{6\pi}{L}} \right) - \left(\frac{\sin 0}{\frac{2\pi}{L}} - \frac{\sin 0}{\frac{6\pi}{L}} \right) \right]$$

$$= \frac{2}{L} \left[(0 - 0) - (0 - 0) \right]$$

$$= 0$$

11.31 Use the $2s$ wave function given in Table 11.2 to calculate the value of r (other than $r = \infty$) at which this wave function becomes zero.

Only the radial portion of the $2s$ wave function depends on r, and ψ becomes zero when $R(r) = 0$.

$$\frac{1}{\sqrt{2a_0^3}}\left(1 - \frac{r}{2a_0}\right)e^{-r/2a_0} = 0$$

This requires (for $r \neq \infty$)

$$1 - \frac{r}{2a_0} = 0$$

$$r = 2a_0 = 2\left(0.529\ \text{Å}\right) = 1.058\ \text{Å}$$

11.33 Explain, in terms of their electron configurations, why Fe^{2+} is more easily oxidized to Fe^{3+} than Mn^{2+} to Mn^{3+}.

The electron configurations of the species being considered are

Fe^{2+}: $[Ar]3d^6$

Fe^{3+}: $[Ar]3d^5$

Mn^{2+}: $[Ar]3d^5$

Mn^{3+}: $[Ar]3d^4$

A half-filled subshell has extra stability. On oxidizing Fe^{2+}, the product has a half-filled d subshell. In oxidizing Mn^{2+}, a half-filled d subshell is being lost, which requires more energy.

11.39 The standard enthalpy of atomization of an element is the energy required to convert 1 mole of an element in its most stable form at 25°C to 1 mole of monatomic gas. Given that the standard enthalpy of atomization for sodium is 108.4 kJ mol^{-1}, calculate the energy in kilojoules required to convert 1 mole of sodium metal at 25°C to 1 mole of gaseous Na^+ ions.

The equation

$$\text{Na}(s) \rightarrow \text{Na}^+(g) + \text{e}^- \qquad \Delta H_1$$

is a sum of the following two processes

$$\text{Na}(s) \rightarrow \text{Na}(g) \qquad \Delta H_2 = 108.4\ \text{kJ mol}^{-1}$$

$$\text{Na}(g) \rightarrow \text{Na}^+(g) + \text{e}^- \quad \Delta H_3$$

Therefore,

$$\Delta H_1 = \Delta H_2 + \Delta H_3$$

ΔH_3 represents the ionization energy of 1 mole of Na(g), which is 495.9 kJ (Table 11.4). Thus, the energy required to convert 1 mole of sodium metal to 1 mole of gaseous Na^+ ions is

$$\Delta H_1 = 108.4\ \text{kJ mol}^{-1} + 495.9\ \text{kJ mol}^{-1} = 604.3\ \text{kJ mol}^{-1}$$

Last question.

Calculate mass of 1 atom of Carbon-12 in grams.

1 mole has a mass of 12.000 grams. 6.022×10^{23} atoms / mole.

$$\text{Mass of 1 atom} = \frac{12.000 \text{ grams}}{6.022 \times 10^{23}} = 1.993 \times 10^{-23} \text{ grams}$$